International Institute for
Applied Systems Analysis
IIASA www.iiasa.ac.at

Application of a Linear Programming Model for Describing a River

Butrimenko, I., Bell, D.E. and Belyaev, I.

IIASA Working Paper

WP-75-116

1975

APPLICATION OF A LINEAR PROGRAMMING MODEL FOR DESCRIBING THE RIVER

I. Butrimenko
D. Bell
I. Belyaev

September 1975 WP-75-116

Application of a Linear Programming Model for Describing the River

I. Butrimenko and D. Bell and I. Belyaev

If we consider the river as the conventional sequence of different points -- city, irrigation areas, dams, tributaries, canals, and parts of the river between every two such points, we can describe the problems we meet by means of the linear programming model.

Constructing the objective function we will try to minimize:

1) The lack of the water in the river for navigational reasons in dry seasons is

$$\sum_{t=1}^{T} \sum_{i=1}^{N_{Fl}} e_{i_1}^{t}$$

where

N flow - the number of the parts of the river as stated above.

T - the number of the periods in the year.

2) The overflows of the water in the river is

$$\sum_{t=1}^{T} \sum_{i=1}^{N_{Fl}} e_{i_2}^{t}$$

3) The shortage of the lower bound for demand for the cities and irrigation areas caused by the lack of

the water in the river is

$$\sum_{t=1}^{T} \sum_{Cj=1}^{N_{CI}} W^t_{Cj_1}$$

where

N_{CI} - the number of the cities and the irrigation areas.

Further, if there are no special remarks, we will consider that all the equations for the cities are appropriate to the description of the irrigation areas.

4) The overflow relative to maximum demand for the cities is

$$\sum_{t=1}^{T} \sum_{Cj=1}^{N_{CI}} W_{Cj_2}$$

Otherwise, to get the objective function we cannot only add together (1)-(4). The disaster effects P caused by the appearance of any of (1)-(4) possibilities or their combination are dependent on the strength of the disaster.

By their consideration, the objective function can be written

$$\text{O.F.} = \sum_{t=1}^{T} \sum_{i=1}^{N_{Fl}} \left(P(e_1) \cdot e^t_{i_1} + P(e_2) \cdot e^t_{i_2} \right)$$

$$+ \sum_{t=1}^{T} \sum_{Cj=1}^{N_{CI}} \left(P(W_1) \cdot W^t_{Cj_1} + P(W_2) \cdot W^t_{Cj_2} \right)$$

where

$P(e_i)$ (or $P(e_2)$) - the disaster effects, caused by the lack of the water (or its overflow) in the river.

$P(W_1)$ (or $P(W_2)$) - the disaster effects, caused by the shortage of the lower (or the overflow relative to maximal) bound of demand for the city.

Optimization factors for the minimizing of O.F. can be:

1. The different, but in given bounds permissible, amount of water taken by the city in the river $d^t_{C_k}$ ($C_K = 1,\ldots,N_{CI}$) in every period t.
2. The amount of water stored in overflow periods $d^t_{D_i}$ (or supplied in dry seasons to the river $r^t_{D_i}$) by dams D_i ($i = 1,\ldots,N_{Dam}$)
3. The amount of water that can be supplied by the canals, where the control is possible.

Now we will describe 1) the cities (including the irrigation areas); 2) the dams; 3) the tributaries; and 4) the canals, with the following equations:

I. The cities (and irrigation areas), which are fed either from the river or canal.

I.1.

$$f^t_l - f^t_m - d^t_{C_K} = 0 \quad (C_K = 1,2,\ldots,N_{CITY})$$

where

f^t_l - the value of the flow before the city C_K in each period t

f^t_m - the value of the flow after the city C_K in period t

$d^t_{C_K}$ - the demand of the water for city C_K in period t.

If

$$f_i^t < \underline{f_i} \quad ,$$

then

$$f_i^t = \underline{f_i} - e_{i_1}^t \qquad (i = 1,\ldots,N_{flow}); \tag{1}$$

if

$$\underline{f_i} \leq f_i^t \leq \overline{f_i} \quad ,$$

then

$$f_i^t = \underline{f_i} + e_i \quad , \quad 0 \leq e_i \leq (\overline{f_i} - \underline{f_i}) \quad ; \tag{2}$$

and if

$$f_i^t > \overline{f_i} \quad ,$$

then

$$f_i^t = \overline{f_i} + e_{i_2}^t \quad , \tag{3}$$

where

$\underline{f_i}$ - lower permissible value of the flow in the river for the navigation reasons.

$\overline{f_i}$ - upper permissible value of the flow in the river for the overflow reasons.

If

$$d_{C_K}^t < \underline{d_{C_K}} \quad ,$$

then

$$d_{C_K}^t = \underline{d_{C_K}} - w_{C_{k_1}}^t \quad ; \tag{4}$$

if

$$\underline{d_{C_K}} \leq d_{C_K}^t \leq \overline{d_{C_K}} \quad ,$$

then

$$d_{C_K}^t = \underline{d_{C_K}} + w_{C_K} \qquad 0 \leq w_{C_K} \leq (\overline{d_{C_K}} - \underline{d_{C_K}}) \tag{5}$$

and if

$$d_{C_K}^t > \overline{d_{C_K}} \quad ,$$

then

$$d_{C_K}^t = \overline{d_{C_K}} + w_{C_{K_2}}^t \qquad w_{C_K} \geq 0 \quad , \tag{6}$$

where

$\underline{d_{C_K}}$ - the minimal amount of the water required by the city.

$\overline{d_{C_K}}$ - the maximal amount of the water that the technical opportunities of the canals permit to transmit.

In the case of irrigation area, the only difference with the city is that $d^t_{C_K}$ ($\overline{d^t_{C_K}}$) are dependent from the time period t. It is caused by the fact, that in winter periods the irrigation area cannot utilize all of the opportunities due to the danger of an ice outburst.

In accordance with the situation on the river the value of the flow f_j can be substituted from (1) - (3) to I.1. In the case of the lack of the water for the navigation reasons, (1) must be used; in the case of the permissible situation, it will be (2) , and in the case of overflow, (3).

In accordance with the situation in the city, the meaning of the city demand must be taken from (4) - (6). It will be (4) in the case of a shortage of the demand, (5) in the permissible situation, and in the case of inadmissible redundance of the demand to the city it is (6).

Evidently, the possibility of different combinations of the cases (1) - (3) for the flow f^t_e with the cases (4) - (6) for the city demand $d^t_{C_K}$ are possible.

The possibility of different combinations for conditions of:

- the flow (equations 1,2, and 3) with
- the status of the city demand (equations 4,5, and 6) are dependent on
- the specificity of the river.

For instance, for Tisza River, the following combinations are possible:

1) The lack of the water for navigation reasons in the river (equation 1) necessarily gives rise to the shortage in the city and irrigation area (equation 4).
2) The normal situation on the river (equation 2) can cause either the shortage (equation 4), or the normal situation (equation 5), or the overflow in the city and in irrigation areas (equation 6).

3) The overflow in the river (equation 3) provides the necessary overflow in the city and irrigation areas (equation 6).

I.2.

$$fi \geq fi_{min} \quad (i = 1,2,.., N_{flow})$$

where

fi_{min} is the permissible value of the flow in the river for sanitary reasons.

I.3.

$$d^t_{C_K} - \Gamma^t_{C_K} - \Delta^t_{C_K} = 0$$

$\Gamma^t_{C_K}$ - the rest of the water after the city C_K available for the river ($\Gamma^t_{C_K} \geq 0$)

$\Delta^t_{C_K}$ - the water wasted by the city C_K

$d^t_{C_K}$ - see I.1.

I.4.

$$f^t_1 + \Gamma^t_{C_K} - f^t_m = 0$$

where

f^t_1 - the meaning of the flow before the discharge by the rest of the city into the river in period t.

f_m^t - the meaning of the flow after the discharge by the rest of the city into the river in period t.

II. The Dams

II.1.

$$f_l^t - d_{Di}^t + \Gamma_{Di}^t - f_m^t = 0$$

where

f_l^t (f_m^t) - the flow in the river or in the canal before (after) the dam in the period t.

d_{Di}^t (Γ_{Di}^t) - the input (the output, or their sum, if there are several outputs) of the dam Di ($Di = 1, \ldots N_{DAM}$) in the period t.

II.2.

$$Q_i^t = Q_i^{t-1} + d_{Di}^t - \Gamma_{Di}^t$$

Q_i^t - the amount of water stored in the dam in period t.

II.3.

$$0 \leq Q_i \leq \overline{Q}_i$$

$\overline{Q}_i$ - maximal amount of water that can be stored in dam Di

III. The Tributaries

$$f_l^t - \sum_{j=1}^{N_T} R_j^t - f_m^t = 0$$

where

f_l^t , f_m^t - see I.1.

R_j^t - the amount of water available for the river from the tributaries.

N_T - the number of tributaries.

IV. The Canals

IV.1.

$$f_l^t + R_{CAN}^t + f_m^t = 0$$

where

f_l^t , f_m^t - see I.1.

R_{CAN}^t - the amount of water available for the river from the canal.

IV.2.

$$R_{CAN}^t < \overline{R}_{CAN}$$

where

$\overline{R}_{CAN}$ - maximal amount of water which can be supplied over the river.